

# Baryons and Pentaquarks in terms of Mesons

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We investigate light meson bound state contributions to six quark Green's functions and establish that the latter has poles corresponding to baryon bound states. We estimate light baryon masses including the Roper resonance. Constituent quark model and the Parton model are natural consequences of this quantum field theoretic picture. We elaborate this analysis to pentaquarks and heptaquarks. An almost model independent prediction for the mass of  $\theta$  pentaquark is 1.57Gev.

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We look for a baryon pole in six quark Green's functions  $\langle q\bar{q}q\bar{q}\bar{q}\bar{q} \rangle$  in QCD. Non-perturbatively this has many contributions, in particular let us concentrate on contributions due to quark-quark scattering. In a relativistic Quantum field theory like QCD this can be inferred from quark-antiquark scattering by crossing symmetry. We assume the color singlet  $s$  channel of  $q\bar{q}$  scattering does have color singlet meson poles which are the observed pion,  $\rho, \omega, K$  etc(Fig[1]). The wave-function  $\phi_n$  of these mesons are realised as eigensolutions of the Bethe-Salpeter( $BS$ ) equation wherein a kernel is subsumed to exist from the underlying theory, QCD.

$$\langle q\bar{q} q\bar{q} \rangle = \sum_n \phi_n^* \frac{1}{s - M_n^2 + i\epsilon} \phi_n + \dots \quad (1)$$

where  $\phi_n$  obeys the  $BS$  equation (Fig[1b]).  $M_n$  is the mass of the meson of type  $n$ .

$$\phi_n = S(p) \bar{S}(\bar{p}) \int K \phi_n \quad (2)$$

$S(p)$  and  $\bar{S}(\bar{p})$  are the exact propagators for the quarks. The  $BS$  wave-function does have a spectral representation which follows from causality[1, 2]. It is given by

$$\phi_n = \int_0^1 dy \int_0^\infty \frac{d\beta \tilde{\phi}_n(y, \beta)}{y p^2 + (1-y) \bar{p}^2 - y(1-y) M_n^2 - \delta_n^2 - \beta + i\epsilon} \quad (3)$$

The spin angular momentum,flavour details are suppressed in the above equation. More precise definitions along with spin-orbit coupling details are presented in

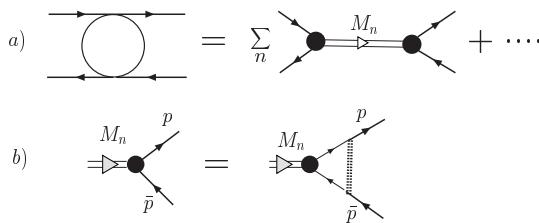


FIG. 1: (a)The Meson contributions to the quark anti-quark scattering kernel. (b) The Bethe-Salpeter equation for the Meson

[2] along with asymptotic behaviour of the  $BS$  wave-function which are deduced from the structure of the  $BS$  equation. For completeness we state that there is a generic spectral representation for the quark propagator satisfying the Schwinger-Dyson( $SD$ ) equation in QCD

$$S(p) = \int_0^\infty d\alpha \frac{\tilde{S}(\alpha)}{p^2 - \tilde{m}^2 - \alpha + i\epsilon} \quad (4)$$

We have introduced two important quantities[3]  $\tilde{m}$  and  $\delta_n$ .  $\tilde{m}$  is the threshold mass *i.e.*, only for momenta  $p^2 \geq \tilde{m}^2$  will the quark propagator have an imaginary part. It may in general be a pole if the quark is not confined or a branch cut if it is confined. Similarly  $\delta_n$  is the relativistic analogue of the size(in units of mass) of the meson[2] of mass  $M_n$ . In particular the meson wave-function  $\phi_n$  is a function of two complex variables,  $p^2$  and  $\bar{p}^2$ . Only for momenta  $p^2 \geq \delta_n^2$  or  $\bar{p}^2 \geq \delta_n^2$  the wave-function  $\phi_n$  has singularities. The threshold mass  $\tilde{m}$  is in principle determined form the  $SD$  equation while the meson size  $\delta_n$  and meson mass  $M_n$  are self-consistently determined from the  $BS$  equations. From eq.(2) it is easy to see that

$$0 \leq \delta_n \leq \min(\tilde{m}, \bar{m}) \quad (5)$$

The lower inequality follows from the the fact that all amplitudes have no singularities in the space-like domain(here it amounts to saying the meson has a finite size) while the upper bound is self-evident from the  $BS$  equation, eq.(2) ( $\tilde{m}$  and  $\bar{m}$  being the threshold masses of the quark and the anti-quark). A more precise estimate of  $\delta_n$  demands the knowledge of the exact structure of the kernel in the  $BS$  equation. From the definition of  $\delta_n$ (eq.(3)), we note that a bound quark in a meson has a threshold singularity at  $\delta_n$  which is less than that of the unbound quark(eq.(5)).

Consider the amplitude of three quarks going to three quarks. We notice that there are contributions due to three different quark-quark scattering. One such contribution of interest is the one shown in Fig[2]. This involves one internal loop momentum  $k$  as explicitly shown in the Fig[2]. By choice we take  $q_i$  and  $r_i$ (with  $r_1 = 0$ ) as independent momenta while the external momenta  $p_i$  and  $\bar{p}_i$  follow from momentum conservation at the vertices. To

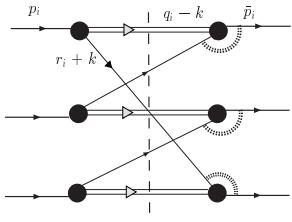


FIG. 2: Baryon pole(constituent quark picture).

avoid the ambiguity of double counting the quark propagator, we introduce the kernel used in the *BS* equation. Using spectral representation for the *BS* wavefunction(eq.(3)) this diagram is completely well defined and the loop momentum, $k$ , integration can be done. We are interested in the singularity structure of this graph. We apply the Hadamard/Landau conditions[4] on all variables being integrated over, which includes  $y_i$  and  $\beta_i$  coming from each of the spectral representations of the meson vertices. To make the computation transparent first notice that we have to consider terms like

$$\Delta = y_i(k + r_i)^2 - (1 - y_i)p_i^2 - y_i(1 - y_i)M_i^2 - \delta_i^2 - \beta_i \quad (6)$$

This is equivalent to an effective mass term for the quark propagator which has momentum  $(k + r_i)$ . Landau equations demand that  $\beta_i = 0$  and  $y_i = 0, 1$  or  $\frac{\partial \Delta}{\partial y_i} = 0$ . Furthermore looking for the lowest effective mass with respect to  $p_i^2$  i.e.,  $\frac{\partial \Delta}{\partial p_i^2} = 0$  we require  $y_i = 1$ . Consequently, we find  $\Delta \approx (k + r_i)^2 - \delta_i^2$ . To evaluate the nature of the singularity it is convenient to employ Cutkosky cuts[5] as shown by the dashed line in Fig[2]. With suitable conditions on  $\tilde{\phi}(y, \beta)$  it yields six Dirac-delta functions which impose the following constraints on the momenta

$$(q_i^0 - k^0) = \sqrt{M_i^2 + (\vec{q}_i - \vec{k})^2} \quad (7)$$

$$(r_i^0 + k^0) = \sqrt{\delta_i^2 + (\vec{r}_i + \vec{k})^2} \quad (8)$$

Four of the Dirac-delta functions will eliminate the loop momentum  $k$  integration completely. One will be the overall pole condition on the total energy(eq.(10)) and the other Dirac-delta function restricts the external momenta. The loop momentum  $k$  integration also yields a self-consistency condition(Landau equation) namely the vanishing of the following four momenta

$$\sum_i \left( -a_i(q_i - k) + b_i(r_i + k) \right) = 0 \quad (9)$$

for some  $a_i, b_i \geq 0$ . The physical interpretation of this equation is that if the masses of all lines in the graph of Fig[2] are taken to be real, then the process depicted in this graph can occur in real space-time classically pro-

vided they obey this constraint[6]. The total energy is

$$E = \sum_{i=1}^3 (q_i^0 + r_i^0) = \sum_{i=1}^3 \sqrt{M_i^2 + (\vec{q}_i - \vec{k})^2} + \sqrt{\delta_i^2 + (\vec{r}_i + \vec{k})^2} \quad (10)$$

Minimising  $E$ , all spatial momenta vanish and we get the lowest baryon mass  $M_B$ ,

$$M_B = \sum_{i=1}^3 (M_i + \delta_i) \quad (11)$$

For this case, the condition in eq.(9) can be satisfied trivially. The mass of the lightest baryon, the proton, is in terms of the mass of the pion taken to be  $M_\pi = 139$  Mev and the size of the pion,  $\delta_\pi$ , inferred to be 174 Mev(1 fm).

The picture that we have deduced from this analysis is precisely that of the constituent quark model. The interpretation of the graph(Fig[2]) is that there is a pole corresponding to a baryon state  $|B\rangle$  and corresponds to the following matrix element,

$$\langle qqq|B\rangle \frac{1}{s - M_B^2 + i\epsilon} \langle B|qqq\rangle \quad (12)$$

where  $s = E^2$  in the rest frame of the Baryon. Consequently we infer the overlap function  $\langle B|qqq\rangle$  in terms of the explicit meson wave-functions. Furthermore for the lightest baryons the three quark momenta  $p_i$  are such that they are constrained to be  $p_i^2 = (313\text{Mev})^2$ . In the rest frame of the baryon these energies of the quarks decompose into masses  $M_\pi$  of the mesons and  $\delta_\pi$  (bound quark mass in the meson) as shown by the cut in Fig[2].

In the theory Fig[2] is not the only pole contribution to the six quark Green's function. Indeed there are many others. Firstly this graph can repeat several times by itself or repeat with other quark propagations in between. All these also have singular Dirac-delta function contributions corresponding to the baryon pole. We envisage a systematic expansion wherein these contributions result in a finite renormalisation of the baryon mass  $M_B$ . Therefore our estimate for  $\delta_\pi = 174$  Mev subsumes these contributions. In 't Hooft's  $\frac{1}{N}$  expansion[7] where  $N$  is the number of colors, we know that every additional meson function implies a factor of  $\frac{1}{\sqrt{N}}$ . Some of these finite renormalisations are controlled in the  $\frac{1}{N}$  expansion. Indeed these contributions makes the evaluation of masses of excited baryons tedious and also detailed knowledge of meson wave-functions becomes necessary.

In addition there are other corrections which allows us to infer Feynman's parton model. For example, using the same technique we can look at Green's functions where the three quark state goes to three quarks and several other gluons and quark anti-quark pairs as shown in Fig[3]. All these graphs also have the same pole contribution as shown by the cut in Fig[3] and indeed this graph

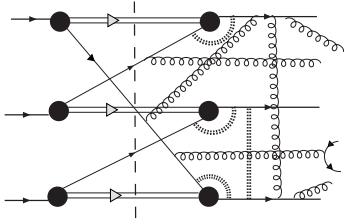


FIG. 3: Baryon pole(Parton picture).

corresponds to

$$\langle qqq|B\rangle \frac{1}{s - M_B^2 + i\epsilon} \langle B|qqq, gg(etc)\rangle \quad (13)$$

where  $g$  stands for gluons. From this we infer  $\langle B|qqq, gg(etc)\rangle$  matrix elements explicitly. That is the baryon state has a finite probability amplitude to produce various other degrees of freedom in addition to the valence quarks.

$$\langle B| = \langle B|qqq\rangle \langle qqq| + \langle B|qqqgg\rangle \langle qqqgg| + \dots \quad (14)$$

From our analysis we see that each term on the the *r.h.s* of the above equation has the same mass eigenvalue  $M_B$  as dictated from Fig[3]. It should be noticed that the constituent quark picture of Fig[2] is disturbed because these constituents can also exchange energy as shown in the Fig[3] leading to parton distribution functions. We also note in the conventional parton picture, quarks with momenta  $\bar{p}_i$ (Fig[2]) emit gluons but we find in general there are quarks with lower energy, of the order of  $\delta_i$  which also emit gluons as shown in Fig[3]. Although formally all these are precisely given in terms of meson wave-functions and quark and gluon propagators, a systematic convergent expansion is an open problem.

Stability of the baryon state would require that  $M_B \leq 3\tilde{m}$ . We have no *a priori* estimate of  $\tilde{m}$  in QCD but a self-consistent picture to be presented later shows that  $\tilde{m} \leq 591 Mev$ . In a model theory such as  $\sigma$ QCD[8] it has been estimated[3] for  $u, d, s$  quarks to be about  $550 Mev$ .

Excited baryons can come about because of excited mesons in the intermediate states of Fig[2]. Note that excited mesons such as  $\rho, \omega$  etc are necessarily unstable because they do decay into  $2\pi$  or  $3\pi$  states. Consequently the meson pole is in the complex energy domain. Our analysis of the Green's functions can be carried out even in the complex plane consistent with causality (*i.e.*, with Feynman  $i\epsilon$  prescription) namely positive energy pole has negative imaginary part and *vice versa*. The external momenta  $p_i$  and  $\bar{p}_i$  then have to be complex. As a consequence the width of the baryon is necessarily greater than the widths of these intermediary mesons. For example if we consider intermediate mesons to be  $\rho$  and  $\pi$  with masses  $M_\rho$  and  $2M_\pi$ , we

get  $M_B = 2(M_\pi + \delta_\pi) + (M_\rho + \delta_\rho) \approx 1474 Mev$  for  $\delta_\rho \approx \frac{\delta_\pi}{2}$  (since the size of the  $\rho$  meson has to be larger than that of the the pion). This matches with the Roper resonance and we would also find that the widths obey  $\Gamma_{Roper} > \Gamma_\rho \approx 150 Mev$ , which is consistent with phenomenology. As we see in this example predicting higher states demands knowing meson properties,  $M_n$  and  $\delta_n$ . The latter being unavailable we have to develop theoretical models. As another example, consider the  $\Lambda$  particle which is made up of  $u, d, s$  quarks. The intermediate mesons in Fig[2] have to be two  $\pi$  and  $K^0$ . From the masses of these mesons we get the mass of the  $\Lambda$  baryon,  $M_\Lambda = 1.2 Gev$  for  $\delta_K \approx \frac{\delta_\pi}{2}$ .

Our method of estimating the spectrum of baryons although unconventional is consistent with quantum field theory. It is also consistent with the Hamiltonian eigenvalue problem. The complication is that the eigenvalue problem is stated in the basis of infinite Fock states. It is interesting to note that the same eigenvalue problem (eq.(14)) can be solved by using Green's functions and their singularities. We can explicitly get the mass eigenvalues and baryon states in terms of meson states. Now, it has to be noted that if we take a system which is essentially non-relativistic, *i.e.*, the threshold masses are large compared to the binding energy,  $b_i$ , then the mass of the 2-body meson bound state is given by,  $M_i \approx 2\tilde{m} - b_i$  and  $\delta_i \approx a\tilde{m}$  where  $a$  is some interaction coupling constant. Consequently we will estimate the baryon mass from Fig[2] to be  $3(2\tilde{m} + a\tilde{m} - b_i)$  which is much greater than  $3\tilde{m}$  (mass of three free quark states) implying that the system is unstable. This shows our procedure is inapplicable for all heavy quark systems. Similar considerations on charm and  $u, d$  quark systems also results in instability for the baryon bound state.

Exact chiral symmetry is another limit where we can draw conclusions. This as discussed in [3] is a singular limit. Due to Goldstone's theorem we have an exact relationship between the pion and the quark propagator [9] namely  $\phi_\pi(p) \propto \gamma_5 Tr S(p)$  showing that  $\delta_\pi = \tilde{m}$  and also  $M_\pi = 0$ . Consequently the mass of the baryon is  $M_B = 3\tilde{m}$  implying that the baryon is almost unstable. Hence in the constituent quark model estimate, the baryon cannot be bound. However there are finite renormalisation effects alluded to earlier. These effects can give rise to a weakly bound baryon state.

Now consider the recently discovered[10] pentaquark candidate which has been discussed in the literature[11]. This is a  $ududs$  color singlet system with mass  $1540 Mev$ . In our picture the graph and cut shown in Fig[4] gives the lowest pentaquark possible. The mass of the pentaquark  $\theta$  is therefore

$$M_\theta = 3(M_\pi + \delta_\pi) + M_K + M_\pi = M_P + M_K + M_\pi = 1571 Mev \quad (15)$$

where  $M_P$  is the mass of the proton or neutron and  $M_K$  is the mass of the  $K$  meson. Interestingly this prediction

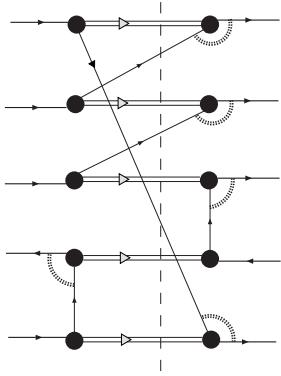


FIG. 4: Pentaquark pole.

does not explicitly depend on  $\delta_\pi$  if the mass of the proton is given. Fig[4] shows that the spin of this state is  $\frac{1}{2}$  as in the case of the proton. As for as the width is concerned, since it is made up of only stable mesons, it is expected to be narrow.

We may look at other pentaquarks such as  $udud\bar{u}$ , our prediction for its mass will be  $M_P + 2M_\pi = 1217\text{Mev}$  and spin  $\frac{1}{2}$ , which is close to the mass of the  $\Delta$  baryon. This leads us to suspect that in the standard  $\Delta$  resonance which has a width of about  $120\text{Mev}$  there exists a pentaquark resonance as well. The standard  $\Delta$  resonance is the  $uuu$  state which in our picture comes about from symmetry considerations in the same way as in the constituent quark model. In terms of graphs, there are actually two graphs of the Fig[2] type and the cancellations between them yields a higher mass to the  $uuu$  system than an  $uud$  system. Similarly the masses of heptaquark states such as  $udus\bar{d}s\bar{u}$  can also be predicted to be  $M_P + 2M_K + 2M_\pi = 2313\text{Mev}$  and this resonance will be a little broader since it has many decay channels.

In our considerations here, the important infrared properties of the quarks and mesons that are relevant to the description of baryons are the threshold masses,  $\tilde{m}$ , meson masses,  $M_n$  and sizes,  $\delta_n$ . We have experimental knowledge of  $M_n$  and to a lesser extent of  $\delta_n$  inferred from form factors. In principle  $\tilde{m}$  and  $\delta_n$  can be inferred from Lattice simulation of QCD. Another attempt is to model QCD  $q\bar{q}$  scattering kernel in the  $BS$  equation(Fig[1b])in a reliable manner.  $\sigma$ QCD[8] is one such approach where the kernel is taken to be purely a one gluon exchange and a singular ' $\frac{g^2}{q^4}$ ' propagator. Here  $q$  is the exchanged momentum between the quark and the anti-quark. This was analysed in the large  $N$  expansion with  $g^2N$ ( $g$  is the gauge coupling constant) also small. In this theory which is asymptotically free, we find spontaneous symmetry breakdown of chiral symmetry and PCAC[2, 3]. The threshold mass  $\tilde{m}$  of  $u, d, s$  quarks is numerically estimated to be  $550\text{Mev}$  and  $\delta_\pi$  can be estimated.

In the above discussion even if the quark is not confined

*i.e.*, the quark propagator has a pole at  $\tilde{m}$ , as long as the inequalities alluded to in the paper are satisfied the results remain true. Color non-singlets such as diquark systems can also be investigated in our picture. The graph of this system is that of Fig[2] but with the middle quark line removed. In 4 dimensions the loop momentum is uniquely fixed and there is no residual Dirac-delta function to restrict the total energy of the Green's function. This implies the diquark system has no pole but only a branch cut singularity. This may have important consequences in scattering processes. Similarly if we take a four quark system, they can be bound like the three quark system with a mass  $4(m_\pi + \delta_\pi)$  even though they can never be a color singlet. Looking at non-singlets one can consider a  $qq\bar{q}$  system which has the same color quantum numbers as a fundamental quark. This yields the same graph as in Fig[4] but without two quarks. The estimate of the mass of this state being  $3m_\pi + \delta_\pi = 591\text{Mev}$  and is necessarily a branch cut. Consequently consistency of our picture demands that the threshold mass be,

$$\tilde{m} \leq 591\text{Mev} \quad (16)$$

Color singlet criterion has played very little role in our analysis, if at all these considerations have to come from other theoretical aspects of QCD. It is assumed that mesons are color singlets. It should be borne in mind that production of these multi-quark states becomes harder because these have to obey the criteria of eq.(9) in the real phase space.

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